THREAT ORDERING CRITERIA AND EVALUATION FOR AN ANTI-BALLISTIC MISSILE DEFENSE

by

Paul Carr Palmer

United States Naval Postgraduate School

THESIS

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Threat Ordering Criteria and Evaluation

For an Anti-Ballistic Missile Defense

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Paul Carr Palmer Jr.
Major, United States Army
B.S., United States Military Academy, 1961

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ABSTRACT

This paper is concerned with those criteria by which enemy ballistic missiles are threat ordered. A questionnaire was sent to Army Air Defense officers at two senior service schools to elicit their opinions. The questionnaires were analyzed using Kendall's coefficient of concordance. Relative weights were determined for each criteria.

A linear model was developed using selected criterion and their relative weights. The number of interceptors was fixed. A sample problem shows how one might use the linear model to determine which ballistic missile to engage first. The number of interceptors to be fired at a threat is discussed and a simple non-linear program is formulated where the number of interceptors can vary.

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ABBREVIATIONS

AADCP - Army Air Defense Command Post

ABM - Anti-Ballistic Missile

AWC - Army War College

BIRDIE - Battery Integration and Radar Display Equipment

BM - Ballistic Missile

C & GS - Command and General Staff College

FOBS - Fractional Orbit Bombardment System

ICBM - Intercontinental Ballistic Missile

MIRV - Multiple independently-targeted Reentry Vehicle

NORAD - North American Air Defense Command

SLBM - Sea-launched Ballistic Missile

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1. INTRODUCTION

The role of Air Defense in the furtherance of national security is as follows:

(1) Detection of potential threat objects

(2) Identification of unknown objects

(3) Interception of enemy forces

(4) Destruction of the hostile threat [Ref. 4, p. 4]

Thus a threat is detected, identified, intercepted, and last destroyed or neutralized. Although these national security objectives are concerned with the aircraft threat, these objectives are no less appropriate to the missile threat.

A. CURRENT SYSTEM

Our current system is designed primarily as a defense against the air-supported threat. The nerve center of the current system is built around the AN/GSG-5 (V), BIRDIE system. This is a fire distribution system which directs the flow of information between the Army Air Defense Command Post(AADCP), individual batteries and the NORAD division direction center.

The necessary control, indicators, and display equipment are located in the AADCP and enable the controller to make or delete target assignments to the firing batteries. But over the last few years this assignment problem has been and will be greatly increased by the advent of the ICBM, the SLBM, the MIRV, and the FOBS. A tremendous research effort has been and is currently studying the different methods of countering these threats.

Even assuming that some type of ABM system is in existance to counter these threats, a number of operational problems pertaining to the four Air Defense roles still remain. This paper will investigate a particular problem that arises in carrying out the interception role of the Air Defense mission.

B. PROBLEM AND SCOPE

The purpose of this paper is to determine criteria by which an enemy ballistic missile threat during an attack can best be measured and to develope a method weighting these factors so that priorities may be established for the destruction of enemy missiles. At this point in time, an enemy threat will likely be made up of a combination of aircraft, both manned and unmanned, ICBM's, MIRV's, FOBS's, and SLBM's. This would be under an all out attack.

A typical attack would most likely be phased. One could first expect nuclear burst's above the atmosphere to create a radar blackout condition. Next would come a simultaneous ballistic missile attack from both the sea and space. Manned aircraft would then follow up the ballistic missile attack.

This paper will address only the threat imposed by a missile attack of the ICBM type. The assumption is made that this type missile will be on a trajectory course and that no evasive action will take place. The problem has been identified as one of "threat ordering and evaluation" by Dr. John O. F. Dorsett. [5] The central theme is that the threats are to be ranked or ordered by some procedure.

The goal of threat ordering is to lower the expected damage from a nuclear attack by efficient allocation of ABM resources. Threat ordering can be motivated by the following simple model. This static model ignores the dynamics of combat, i. e., it looks at a nuclear attack during the whole time of the battle.

$$\min \sum_{i=1}^{N_m} D_i(1 - P_s)^{n_i}$$

$$\text{subj. to } \sum_{i=1}^{N_m} n_i = M$$

$$M \geq N_m$$

$$n_i \text{ integer}$$

where

N_m = total number of enemy ballistic missiles

D_i = damage that i - th BM can produce

P_s = single shot kill probability

n_i = number of interceptors that are to be fired at the i - th BM.

Although this model is simplified and will not be explicitly solved, it does clearly indicate that there should be at least one interceptor fired at each BM. Examination of the model shows that there are two ways to increase system effectiveness, i. e., reduce expected damage. First, one could try to increase P_s. But P_s is expected to be quite high at the start, say at least above .90. Thus any marginal increase in P_s would most likely not be offset by the marginal cost of obtaining the increase. The second area

of interest would be to increase M, i.e., have a large number of interceptors available. The assumption made in this paper is that M is two or three times greater than N_m.

Thus one sees that an effective ABM system would probably be characterized as having a large number of interceptors in relation to estimated enemy missile strengths. Unfortunately, the number of interceptors that can be simultaneously deployed is limited by the number of interceptor control radars. A very complicated allocation problem then arises. Allocation here refers to the proper use of the interceptor control radar's time. This is essentially a sequencing problem. Threat ordering is one method of determining priorities for the deployment of interceptors.

As a first pass in looking at this sequencing problem, the allocation of interceptors to various BM's during a specific time period within an attack will be examined. A raid band, to be defined later, will be the time period used in the nonlinear programming model to be developed.

More precisely the problem this paper will examine is that within the framework of a ballistic missile attack:

(1) What criteria should be used to order the threats?

(2) What weight should be assigned to each criteria?

One can start by looking at the overall picture and then scaling it down to a workable size. The ideas that pertain to just one site or complex are essentially just as valid as when applied to the whole

system. The passing of information between defended areas is an intergral part of this overall problem, but will only be discussed when it is of importance to the above defined problem.

This passing of information is a command and control problem, but the important information would be that pertaining to threat ordering and would normally be made up from radar inputs. Also those defenses first encountered by an attack may have a more difficult task in so far as they must determine which ballistic missiles they must shoot down and which they can afford to let pass into subsequent defended areas. Those defenses at the termination of a ballistic trajectory will not really be concerned with this problem, since they must shoot down every incoming missile.

We can consider ABM complexes located throughout the country. An example is shown in Figure 1. Each circle represents a defended area consisting of several radars, a control and computer center, and missile batteries. For this problem, our discussion will be based on just one of these defended areas. Thus all ballistic missiles whose trajectories are predicted to impact in say, area A, will concern us. This could mean that the theortical maximum capability that the defense could be expected to have would be the destruction of all threats both passing through the defended area and those intended for the area. But those passing over the area would essentially be attacked in the same manner as those predicted to land in the area after all the latter have been destroyed.

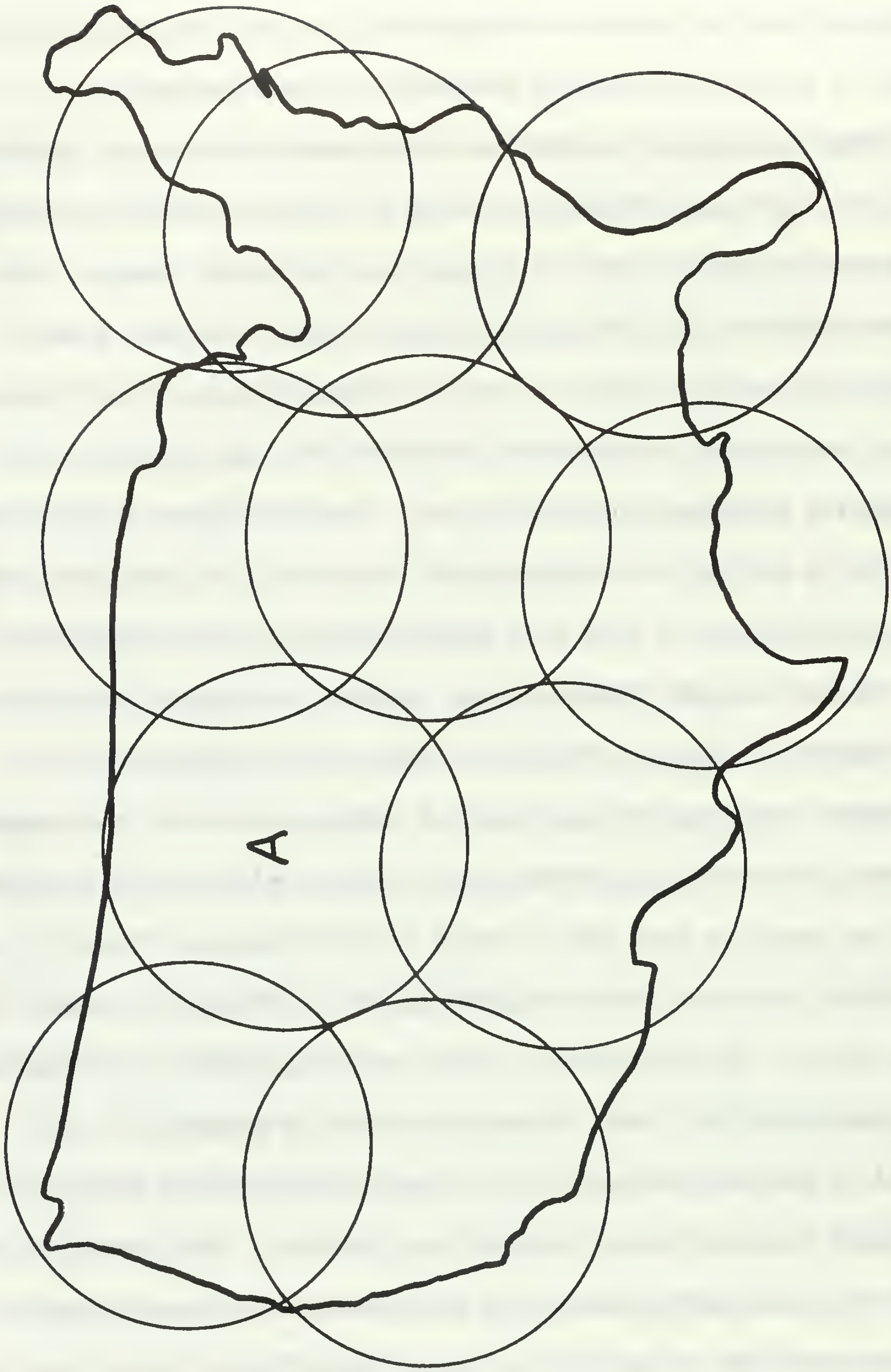

Figure 1. ABM COMPLEXES

Threat ordering consists of looking at all the incoming missiles and determining which one constitutes the largest threat and then taking a defensive action, e.g., fire two missiles. To accomplish this threat ordering, the determination of the criteria and the relevant weight of these criteria is required before the threat can be ranked as to the highest threat, next highest, etc.

But one may ask the question, "Why take time to rank them at all when you come under attack? Why not just shoot them down on a first come, first serve basis?" This may seem quite plausible at first and is satisfactory when the threat is made up wholly of aircraft. In this situation there is time available in which to make a decision, look at the probable results of that decision, and alter the decision if necessary.

Now when a missile threat is considered, this precious time is lost. It is of utmost importance that threat ordering be done rapidly and accurately because of the short warning times. An automated system with a near real time processing capability is required. Any so-called decision making by the computer will require that the most important criteria and their relative weights be automatically incorporated into our defensive system at each moment of time. A controller will not have time to point at a radar scope and direct fire units at a specific threat or even direct that the semi-automatic fire distribution system (AN/TSQ051) perform this duty. The firing doctrine, however simple, will have to be automatic.

Another important consideration that certainly warrants any time spent on this problem does not concern the defensive capability, but that of the offensive force. It is feasible to consider the idea that the ABM system could serve as an input into our own ICBM network and on evaluation of threat provide the time to fire our own ICBM's. In other words, the ABM system would serve as an additional warning net to our offensive forces. Analysis of missile threat ordering would be a prime function.

C. PLAN OF STUDY

The plan of this study is to first find out which criteria are important and which should be used. Next a questionnarie containing the criteria will be sent to the Air Defense Officers at the Army War College and the Army Command and General Staff College where selected criteria will be ranked. The questionnaires will then be analyzed using Kendall's coefficient of concordance to determine the degree of agreement concerning the criterial. A few simple threat functions will be discussed. A sample problem will be developed showing how these functions can be used by the computer to order ballistic missile threats.

II. SELECTION OF THREAT CRITERIA AND WEIGHTS

As already alluded to one might be quick to say that the closest missile to the defended area should be fired at first. But what if the one just following has a larger yield and/or is directed at a higher priority target? A problem then exists as to which should be destroyed first in order to minimize overall damage. Also in this case, if there were only a few missiles remaining or another wave were expected, one would most likely want to fire at the larger yield, higher priority type BM's first. Thus more than one criteria need be used to determine the greatest threat. Another way of stating this is that one is interested in finding out what procedure to use for allocating interceptors to incoming enemy BM that will minimize the damage to the defended area and/or the country as a whole.

A. CRITERIA SELECTION

The first method looked at to determine how to arrive at the proper threat criteria was suggested by Churchman. [Ref. 3, p. 136-153]. This method makes use of criteria ranking by a panel of experts. The selection of the initial criteria to be looked at is the result of the considered opinion of a responsible panel. The panel is asked to make an initial ranking of the criteria. This is done on an individual basis. Then a series of comparisons is made by the panel as a whole. The use of group opinion in the

ranking method is accomplished by accepting a majority rule on each comparison and regarding the collective decision as a single response. Then the criteria included are specifically defined to provide for equivalent comparisons among the members of the panel. Decisions are made by open ballot, but discussion of the specific responses is prohibited. The periodic adjustment conforming to each vote is performed in full view of the participating panel.

This method in its original form was not suitable for this study, since assembling a panel of experts was not possible. The questionnaire method was used. The questionnaire is shown in Appendix A. The criteria have been written for the general case and not for any particular scenario.

The first criteria on, denoted C1, has the effect of predicting the yield of the BM. As the analysis of radar cross-section is well developed, [Ref. 1, p. 51] a rough estimate can be made of the BM's yield. From the yield a determination of how large an area will be destroyed can be made. For example, if the radar picks up a BM with a cross-sectional area of .5 meters, then it might be estimated to be a 50 kiloton warhead, and thus have the ability to destroy an area of 10 square miles.

The second criterion, the BM's range from the defended area, denoted C2, is concerned with the range from the closest firing unit within the defended area. Another equivilant meaning is the distance in time, say in seconds, before the BM would impact in the defended area.

The third criterion is the value of the impact area to the United States and is denoted C3.

To make use of the impact criteria, the idea of isocost or value contour lines are introduced. These lines will be used to high light those areas of the greatest population concentration, industrial areas, and offensive missile locations. These contour lines would be used to give two or three rough orders of magnitude as to the loss that would be incured if a missile hit that particular area. For example, look at the defended area A in figure 2.

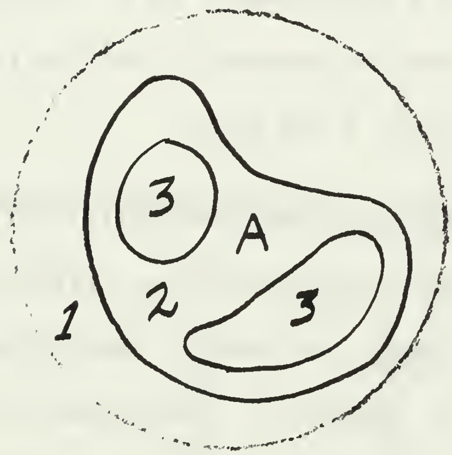

Figure 2. Single ABM Complex

The area having a value of 1 would be equal to 1 million people or have a property value[1] of 100 milion dollars. Area 2 would be worth twice this amount and area 3 would be three times this. These

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[1]This value can also be interpreted on a strategic level. Of strategic value to the United States would be the types of skills possessed by the people in an area. This is vital for the regeneration of our system in a post-nuclear strike environment. It is important that a "balanced profile" of survivor skills be present after a nuclear exchange. Stanford Research Institute and the RAND Corporation have done studies along these lines.

lines could be put into a computer routine, so that they could be changed rapidly when a change in value was noted, such as after our ICBM's were launched or a sizeable amount of the population had moved out of a city. These values will most certainly help in ordering BM threats. The criteria is denoted C4.

The last criteria concerning the number of batteries is also self-explanatory and is denoted C5.

The questionnaires were sent out to the aformentioned Army schools. After the questionnaires were returned, the 5 weights were ranked on a scale from 1 to 5. The results of the original weights are shown in Appendix C and the results in a ranked form are shown in Table 1 and 2.

B. RANKING PROCEDURE AND HYPOTHESIS TESTING

In order to determine how the criteria should be ranked, and if they can be ranked, the Kendall coefficient of concordance was used. [Ref. 10, p229-239]. The degree of agreement among the k judges is reflected by the degree of variance among the n sum of ranks. W, the coefficient of concordance, is a function of that degree of variance.

The method is to first find the sum of ranks, R_j, in each column of the k by n table. Then W is computed by

$$W = \frac{S}{1/_{12}K^2(N^3-N)}$$

where s = sum of squares of the observed deviations from the mean of R_j, such that

TABLE 1

RANKED THREAT DATA FOR AWC

Judge	C1 (yield)	C2 (time)	C3 (# of inter)	C4 (Impact)	C5 (# of batt)
1	4	1.5	3	5	1.5
2	1	4	3	5	2
3	3	4	1.5	5	1.5
4	3	4	1.5	5	1.5
5	1	2	5	3	4
6	1	5	3	4	2
7	4	5	3	2	1
8	4	5	2	3	1
9	5	3	1.5	4	1.5
Sum	26	33.5	23.5	36	16

TABLE 2

RANKED THREAT DATA FOR C & GS

Judge	C1	C2	C3	C4	C5
1	2	1	5	3	4
2	1	3	4.5	4.5	2
3	1	3	5	2	4
4	3	1	5	2	4
5	3	5	2	4	1
6	1	5	3	4	2
7	1	4	3	5	2
8	1	5	3	4	2
9	3	5	4	2	1
10	1	5	3	4	2
11	4	3	2	5	1
12	5	3	1.5	4	1.5
13	1	4.5	3	4.5	2
14	3	4	2	5	1
15	3	5	1	4	2
Sum	33	56.5	47.5	57	31.5

$$s = \Sigma\left(R_j - \frac{\Sigma R_j}{N}\right)^2$$

k = number of judges

N = number of criteria ranked, i.e., 5

$\frac{1}{12} K^2(N^3-N)$ = the sums which would occur with perfect agreement among $\underline{k}$ rankings

The results are shown in table 3. For example, let us

TABLE 3

RESULTS OF THREAT ANALYSIS USING
KENDALL'S COEFFICIENT OF CONCORDANCE

	W	level H_0 rejected	Criteria ranked in order
Army War College	.324	.05	C4,C2,C3,C4,C5
Command & G S	.271	.01	C4,C2,C3,C1,C5
Pooled Data	.271	.001	C4,C2,C3,C1,C5

examine the results from the Army War College. Since the observed value of the test statistic, i.e. W, is greater than the critical value for α = 0.05, one can conclude that the agreement among the 9 judges is higher than it would be by chance. The probability under the null hypothesis (H_0) associated with the observed value of W enables one to reject the H_0, that the judges ratings are unrelated to each other. Here W = .324 expresses the degree of agreement among the 9 judges in ranking the 5 criteria.

Siegel states that a significant value of W can be interpreted as meaning that the judges are applying essentially the same standard in ranking the N objects under study. W has a range from 0 to 1,

where 1 would mean perfect agreement. Siegel goes on to state that often their pooled ordering may serve as a "standard", especially when there is no relevant external criteria for ordering the objects. [Ref. 10, p 238]. It should be noted that a significant value of W does not mean that the orderings observed are correct but merely that the judges agree. It is possible that a variety of judges can agree in ordering criteria because they use the "wrong" reasons. This, of course, is a critical problem and is what really makes it difficult in assigning proper weights to the criterion.

For this study, if one accepts the criteria which the various judges have agreed upon in ranking the 5 entities, then the best estimate of the "true" ranking of those entities according to that criteria is provided by the order of the sums of ranks.

In all three samples, C4 was ranked first. C2 was ranked second. Also the number of batteries able to fire at the BM, C5, ranked last. A problem arose in the ranking of the two criteria, C1 and C3. From the data received these two have almost equal ranking. In the pooled data from the Army War College and the Command and General Staff College, C3 outranks C1. But this nearly equal ranking would seem to imply that more knowledge is really required before the criteria can be ranked. If a scenario were written for different type attacks, then it most certainly would be easier to rank the criteria.

In the next chapter where threat functions are developed the first through the fourth criterion, C1, C2, C3, and C4 will be used. After examining the criteria more closely and the written comments, certainly the number of batteries able to fire at a BM, C5, is related to the number of interceptors that we can launch. This is now realized as a poor criteria.

Thus the criteria and their ranking to be used in the remainder of this paper are shown in Table 4. It is felt that these are the

TABLE 4

THREAT CRITERIA

C1	Yield of weapon(warhead size as determined by radar cross section)
C2	BM's range from the defended area of time to impact
C3	Number of interceptors that we can launch at a BM before it detonates
C4	Predicted point of BM detonation

major criteria that should be used in ranking an incoming BM attack.

Combining the data from both the AWC and the C & GS, the contribution that each criterion makes to the total can be determined. This is accomplished by dividing the sum of each particular criteria by the grand total of all columns, 312.5. For example the yield, C1, contributes 59/312.5 or .19 of the threat weight. The others are C2 = .29, C3 = .22, and C4 = .30.

Some of the major criteria that the officers added to the questionnaires note mentioning. The criterion listed most often was the value of the target under attack. But this would be considered to be the same as the predicted point of BM detonation. The total number of BM's was also listed and it would appear that this might be of some importance. The number of interceptors that could be controlled at any one time is related to the total number of BM's that would have to be evaluated. It is reasonable to assume that the process of evaluating, say 8 BM would take more than twice as much time as evaluating only 4 BM.

The predicted point of intercept is another criterion listed. This would be used to make burst locator diagrams which would be helpful in the placement of batteries. But once the batteries have been placed, then it is the number of interceptors that can be fired that is really important.

III. THREAT FUNCTIONS

Once criteria have been selected and the relative weights have been assigned, they must be combined in some type of mathematical model. This is so that the BM of greatest threat can be determined,then the BM of the next greatest threat, etc.

A. LINEAR MODEL

The first method and the simpliest threat weight function (TW) is a linear combination of the criteria and its associated value,

$$TW(j) = \sum_{i=1}^{4} C_i \quad V_i \qquad (1)$$

where $C1 = .19$

$C2 = .29$

$C3 = .22$

$C4 = .30$

$V1$ = value of yield in kilotons

$V2$ = value of time before impact in seconds

$V3$ = value of no. of interceptors to be fired

$V4$ = value of predicted point of impact

$TW(j)$ = threat weight for the j-th BM

Now this threat weight has meaning only for one defense area and not for any individual battery. That ballistic missile determined to have the highest threat would then be fired upon first. In order to have the total threat weight value make sense,

some of the individual weights have to be negative, i.e., they have the effect of downgrading the threat. For example if more interceptors are fired at an incoming BM, the expected damage would decrease.

Now it is assumed that a computer would be evaluating the BM's threat continually in real time. Thus the data, such as the yield and the predicted impact would be continually updated. Only those BM within the range of the interceptors would be threat ordered, but of course all BM would be tracked.

Then once the greatest threat is determined, it will be engaged. The computer will again select the greatest threat from those remaining. The order may have changed of those remaining BM because of the updated data. Some of this updated information could be coming from other defense complexes or Air Force radars.

Equation (1) in its present form is unsatisfactory, since it is dimensionally inhomogenous, i.e., combines kilotons with time until impact. This may be remedied by normalizing the threat values for the four criteria to a scale on which the value for each criterion is a number between 0 and 1. There is no unique way of doing this. A linear scale on which the minimum value of threat corresponds to 0 and the maximum to 1 seems reasonable as a first attempt. If the values are scaled in this manner, the equation is,

$$TW(j) = \sum_{i=1}^{4} C_i \left(\frac{Vi - V_i min}{V_i max - V_i min}\right) \qquad (2)$$

where Ci = same as equation 1

V_i = same as equation 1

$V_i min$ = minimum value of the V_i,s

$V_i max$ = maximum value of the V_i,s

TW(j) = threat weight for the j-th BM.

In order for these values to make sense when summed, a sign convention must be used. Specifically the predicted impact value and yield will be entered as positive values and the time to impact and number of missiles to be fired will have negative weights.

Now if it is not desirable to use the number of missiles available to fire in determining the largest threat, equation (2) can be used with the missile criteria deleted, and then readjust the remaining criteria. These readjusted values are:

C1 = yield of BM, .25

C2 = time to impact, .37

C3 = impact point, .38

It is noted these criteria values will always add up to 1.00. Once the largest threat has been determined, the number of missiles needed to fire can be determined and thresholds can be set up to allocate these interceptor resourses. More concerning this area will be discussed in the next section.

If this method is used, a threat weight equation in explicit form is,

$$TW(j) = .25V_1 - .37V_2 + .38V_3 \qquad (3)$$

Where V_i is scaled on a 0 to 1 interval as in equation (2)

$TW(j)$ = threat weight of the j-th BM.

In the next chapter a sample problem is shown. It shows how these equations might be used to threat order BM's with the aid of a computer.

B. INTERCEPTORS TO BE FIRED

The determination of how many missiles are to be fired is a critical area of threat ordering. It would be reasonable to say that a BM is too destructive to contemplate letting any come through the defense. This implies shooting at least one ABM interceptor at every BM if at all possible. If each missile has a single shot kill probability of P_s, then one can determine how many missiles must be fired to attain a specific protection level. In probability terms this would be,

$$P_t = 1 - (1 - P_s)^N$$

where

P_t = probability BM will not penetrate defense

P_s = single shot kill probability

N = no. of interceptors fired

The reader is reminded that this equation is valid when the effects of each interceptor are independent. Thus, for example,

if P_s = .90, two interceptors must be fired to attain a probability of .99 that the defense will not be penetrated. A system with a .95 to .99 effectiveness would be considered a practical objective.

With a P_s of .90, two interceptors would have to be fired to obtain a .99 coverage. But this does not necessarily mean that two interceptors would have to be fired at the same time. If time allows, the first interceptor can be launched, the results noted and then another launched if required. The defense would know that this one remaining interceptor was allocated for that BM until either used or found to be not needed. Also it might turn out that one or two more interceptors have to be fired if there is still time and the threat caused by that BM is still large enough.

If one even assumes a P_s of .95, two interceptors are required to attain an effective coverage of .99. So the number of interceptors to be fired at each BM will be set at two for the sample problem in the next chapter and will remain constant.

As an extension to having a minimum of two interceptors allocated for each BM, this rule might have to change under the following conditions. Suppose intelligence has told us that we can expect to receive 40 BM's attacking the defended area. If upon attack, only 20 BM's are detected, then cause would exist to possibly expect a second wave. In a case such as this the defense may not decide to shoot at each BM. Those predicted to land in sparsely populated and/or low valued areas would be let through

so as to be sure interceptors were still available for a possible second wave. This type of consideration will not be pursued further, but it does seem worthy of future research.

One may feel that an assignment of two interceptors to each BM is too arbitrary. Recall that a BM is too destructive to allow any to get through and there is evidence that three interceptors will be available for each BM. [Ref. 1, p. 44]. But it is apparent that a better defensive job could be done if the interceptors were allocated with some type of cost or priority in mind. Now recall that it has been assumed there are enough interceptors and that the control radars are the real constraining factors. So the problem is that given a certain number of control radars, how should they be used to control interceptors against an attack in such a way that damage will be minimized?

The same four basic criteria may be used in a model for the allocation of interceptors and control radars. The criterion time to impact is used to place the incoming BM's into so-called raid points or, more appropriately, raid bands. Raid bands are time zones of equal length where the time means time to impact for a BM. For example, all those BM's 100 to 150 seconds from impact would be threat ordered first, if this was the first time zone in which BM's were encountered and the zone has a length of 50 seconds. Then each BM would be assigned so many interceptors and fired upon. Next those in the interval 150 to 200 seconds from impact would be considered as raid band 2 and the same

procedure as in previous raid band would be accomplished. By this raid band method, the entire attack is reordered as to impact time. Of course it is realized some time would elapse while the first raid point is being evaluated so the BM's would be continually changing from time interval to time interval causing the threats to be changing in each interval. The determination of such a dynamic programming problem would certainly be an appropriate area for further research as the threat criteria values determined in Chapter II remain valid.

Short of developing a dynamic programming solution the idea can be shown in the following non-linear programming formulation. The number of interceptors available is fixed and it is assumed that sufficient interceptors are available. This problem is constrained by the total number of interceptors that can be controlled by the radars at any one time. So to formulate this problem, let the i-th BM be capable of Damage D_i. The formulation is,

$$\min \sum_{i=1}^{N_m} D_i (1 - P_s)^{N_i}$$

$$\text{subj to } \sum_{i=1}^{N_m} n_i \leq CR$$

$$1 \leq n_i \leq 3$$

$$n_i \text{ integer}$$

where

N_m = total number of BM's

n_i = number of interceptors to be fired at the i-th BM

CR = total number of control radars available

D_i = warhead yield times the value of the impact point for the i-th BM

P_s = single shot kill probability.

The total number of interceptors available would most likely be a very loose constraint. A lower bound of 1 is placed on n_i for the same previously stated reasons and the upper bound of 3 is arbitrary but of course would not be unlimited. This upper bound depends upon CR and P_s. It should be noted that this program is static and would essentially allocate interceptors within a specific raid band.

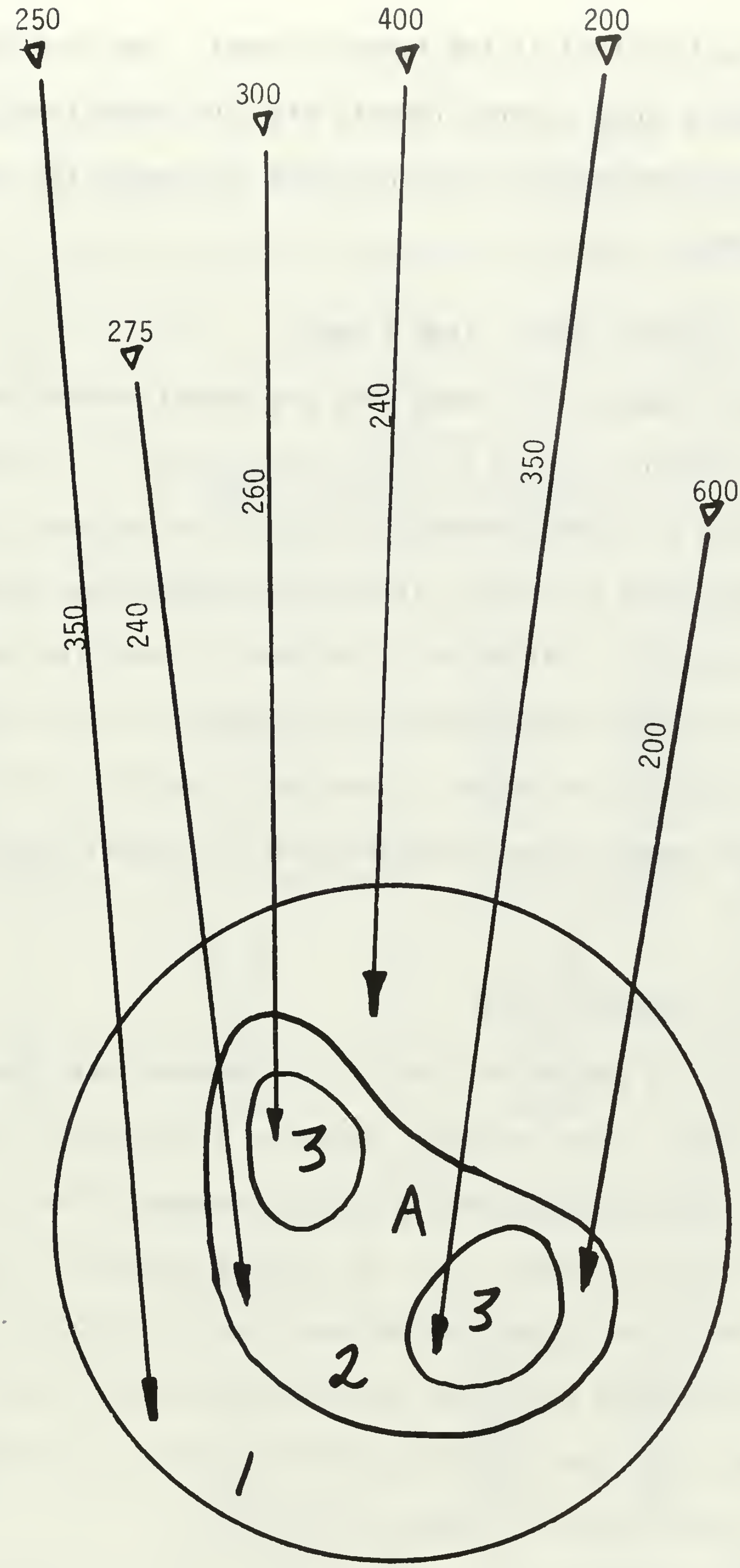
IV. SAMPLE PROBLEM

A sample problem will now be presented. This sample problem shows how one might apply the preceding methodology using a digital computer. The linear model is used. As this program has been written, thirty BM's can be ordered. But one need only change the dimension statement in the computer program and any number can be ordered. To allow one to follow the computer program, see Appendix B and the computer program.

Equation 1 and 3 are used and also compared. Since Equation 1 is summing unlike quantities, the magnitude of any particular piece of data can completely negate another smaller quantity. In other words if the yield had of been in megatons instead of kilotons or the time in minutes instead of seconds, the results would have been quite different.

Three criteria are used, i.e., C1, yield of BM in kilotons; C2, time to impact in seconds; and C3, value of the impact point. This example considers six BM's and is depicted in figure 3.

As each BM is detected, the values of C1, C2, and C3 would be input to the computer in real time. Since equation 3 normalizes the data, it would first have to be sorted in order to get the range of the yields, time, and impact values, V1, V2, and V3 respectively. Equation 1 and 3 are then computed and the values are shown associated with their input data. The weights are then



Legend

400 ▽ - 400 Kiloton Warhead

350 → - 350 Seconds until Impact

Sketch of Sample Program

Figure 3

reordered so that BM 1 is the largest threat. One then need only associate these ordered threats with the respective unordered weights to determine which data was used to compute the value of the equations.

A. CASE I, VARYING YIELD, TIME & VALUE

The data labeled with these BM's are threat ordered and the results are shown in Table 5. Thus for equation 1, the data corresponding to a 600 kiloton yield, a 200 second predicted time to impact and an impact value of 2 is determined to be the largest threat. In this example the same is true using equation 3. For the second largest threat using equation 1, the time factor is the decisive factor. Equation 3 considers both the time and the impact value in determining the second largest threat.

B. CASE II, CONSTANT YIELD

In this case the yields are all the same and have been set at 500 kilotons. (See Table 6). Equation 1 determines that the BM with a time of 200 seconds to be the greatest threat. Next when the times are equal, i.e., 240 seconds, equation 1 then looks at the V3 value and picks that BM predicted to hit an area with V3 = 2. Thus when the yields are the same, equation 1 takes the closest BM as the greatest threat and only looks at the value of the impact point when the times are the same.

Equation 3 determines the greatest threat to have a time of 260 seconds and predicted to hit an area of value 3. Then equation 3 drops to that BM which is 200 seconds from impact. Equation 3 is almost equally concerned with where the BM is as where it will land.

TABLE 5

THREAT ORDERING WITH VARYING YIELD, TIME, AND VALUE

VALUE EQ. 1	VALUE EQ. 3	V1	V2	V3
11.58	0.03	400.	240.	1.
76.76	0.44	600.	200.	2.
-19.29	0.14	275.	240.	2.
-20.06	0.29	300.	260.	3.
-78.36	0.01	200.	350.	3.
-66.62	-0.34	250.	350.	1.

PRIORITY	EQ. 1 ORDERED	BM	EQ. 3 ORDERED	BM
1	76.76	2	0.44	2
2	11.58	1	0.29	4
3	-19.29	3	0.14	3
4	-20.06	4	0.03	1
5	-66.62	6	0.01	5
6	-78.36	5	-0.34	6

TABLE 6

THREAT ORDERING WITH CONSTANT YIELD

VALUE EQ. 1	VALUE EQ. 3	V1	V2	V3
36.58	-0.10	500.	240.	1.
51.76	0.19	500.	200.	2.
36.96	0.09	500.	240.	2.
29.94	0.23	500.	260.	3.
-3.36	0.01	500.	350.	3.
-4.12	-0.37	500.	350.	1.

PRIORITY	EQ. 1 ORDERED	BM	EQ. 3 ORDERED	BM
1	51.76	2	0.23	4
2	36.96	3	0.19	2
3	36.58	1	0.09	3
4	29.94	4	0.01	5
5	-3.36	5	-0.10	1
6	-4.12	6	-0.37	6

C. CASE III, CONSTANT TIME UNTIL IMPACT

In this case the time until impact is constant and is set at 360 seconds. (see Table 7). As might be expected equation 1 orders the threats according to yield completely as the values for the impact area is insignificant.

Equation 3 just by coincidence orders 2 BM's as the same threat because of the range of V1 and V3 and the weights of criteria C1 and C3 are averaged out by the criteria values. When the times are equal one sees that the values of the impact points are the dominating factors.

D. CASE IV, CONSTANT IMPACT VALUE

For this case the impact values were all set to 3. (see Table 8). Since the magnitude of V1 and V2 are about the same, equation 1 and 3 order the BM in the same manner in this case. The time to impact is the major factor in this case.

It is of interest to note that when the threat weight (TW) as determined by either equation is positive that the BM's yield and impact value are the deciding factor, while a negative value implies that the time to impact is the major factor.

It is apparent from the four cases that equation 3 is far superior to equation 1 and should be used for command and control purposes.

TABLE 7

THREAT ORDERING WITH CONSTANT TIME UNTIL IMPACT

VALUE EQ. 1	VALUE EQ. 3	V1	V2	V3
-32.82	0.13	400.	360.	1.
17.56	0.44	600.	360.	2.
-63.69	0.24	275.	360.	2.
-57.06	0.44	300.	360.	3.
-82.06	0.38	200.	360.	3.
-70.32	0.03	250.	360.	1.

PRIORITY	EQ. 1 ORDERED	BM	EQ. 3 ORDERED	BM
1	17.56	2	0.44	2
2	-32.82	1	0.44	4
3	-57.06	4	0.38	5
4	-63.69	3	0.24	3
5	-70.32	6	0.13	1
6	-82.06	5	0.03	6

TABLE 8

THREAT ORDERING WITH CONSTANT IMPACT VALUE

VALUE EQ. 1	VALUE EQ. 3	V1	V2	V3
12.34	0.03	400.	240.	3.
77.14	0.25	600.	200.	3.
-18.91	-0.05	275.	240.	3.
-20.06	-0.09	300.	260.	3.
-78.36	-0.37	200.	350.	3.
-65.86	-0.34	250.	350.	3.

PRIORITY	EQ. 1 ORDERED	BM	EQ. 3 ORDERED	BM
1	77.14	2	0.25	2
2	12.34	1	0.03	1
3	-18.91	3	-0.05	3
4	-20.06	4	-0.09	4
5	-65.86	6	-0.34	6
6	-78.36	5	-0.37	5

V. CONCLUSIONS AND EXTENSIONS

The problem of how can an ABM system minimize damage from an enemy nuclear strike has been examined. The characteristics of an effective defensive system have been identified as having:

(1) Many more interceptors than enemy missile capability

(2) As many control radars as possible

(3) High system effectiveness (probability of intercept) per individual missile

This study has shown that even though there are many move interceptors than enemy missiles, all defensive resources cannot be simultaneously deployed due to the constraints of interceptor guidance systems (control radars). Thus, priorities must be established for the engagement of incoming enemy missiles.

Those criteria found to be the most important in determining which ballistic missile to fire at are the yield of the warhead, the predicted impact point, and the time remaining until the ballistic missile impacts.

Since the overall objective for wanting to threat order ballistic missiles is to minimize the expected damage in an attack, the allocation of interceptors plays an important part. There are a number of methods by which this may be done, two of which have been discussed. In the first method, the ballistic missile's are threat ordered according to yield, time to impact, and impact point. Then a fixed number interceptors, e.g., two, are fired at

each ballistic missile according to its ordered threat weight. The underlying assumption is that not enough missile control radars or control units exist to fire at all ballistic missiles at once. As soon as a control radar is freed, it takes on the largest threat still unengaged.

For the second method, a non-linear programming problem with integer constraints was formulated. Again the problem is constrained by the number of control radars. But this method is different from the first one in that the number of interceptors is not fixed. Expected damage is still minimized using the same criteria as in the first method. A damage coefficient is computed by multiplying the yield of the warhead times the impact point value. The time before impact is used to place the ballistic missile in so-called raid points using time intervals, e. g., 150-200 seconds.

There are almost an unlimited number of extensions that one might look at in the area of threat ordering. The most obvious would be to subdivide the general situation as discussed in this paper into specific scenarios. This would mean looking at those defensive areas located on the boundaries and those in the interior. This would mean looking at two or more defensive areas together and determining how best to coordinate the effort for the overall defense of the country.

One could also extend this thesis to see what criteria and weights should be used when the attack consists of not only ICBM's, but MIRV's, FOBS's, and SLBM's.

Although time did not permit, it was the author's intention to develope a threat subroutine written in Fortran to the extent that it would be used in a simulation model. The Mixed Air Battle Simulation (MABS), version IV, was obtained from Stanford Research Institute. [Ref. 11]. This simulates simultaneous surface-to-air missiles, antiaircraft artillery, and interceptor defense operations. Specifically, 100 ballistics missiles can be simulated. A threat routine is present in this simulation, but is mainly concerned with the aircraft threat. It should prove of interest to compare the original simulation with one in which a threat routine is used specifically for ballistic missiles.

APPENDIX A

THREAT ORDERING QUESTIONNAIRE

Maj. Paul C. Palmer Jr.
Air Defense Arty
Naval Postgraduate School
Monterey, California

Background

The threat constituted by enemy aircraft is such that time is usually available to analyze the attack as it is taking place. A misjudgment on the part of a controller can usually be corrected without greatly degrading the defense. With the threat of a ballistic missile (BM) attack, however, this valuable time is lost.

Picture the following situation. A group of BM's are approaching in a tight formation, but not so tight that our defense can destroy the group with one ABM. We must determine an order by which the BM's are intercepted. This problem is known as "threat ordering".

Threat ordering is defined as the determination of criteria and the relevant weight of these criteria so that a threat can be ranked as to the highest threat, next highest, etc. Once the most important criteria are determined, weights must be assigned. It is then possible to relate these different criteria and weights in some functional form. Our overall defense doctrine would then be to intercept that BM having the highest value.

The determination of the best criteria to use and their weights are partly subjective. In a thesis I am preparing these threat criteria and weights are required. Your opinion is requested in the following areas on the next page.

A. The following criteria have been listed in a random order. You are to assign weights between 0 and 100 in such a manner that the criteria are ranked.

Criteria	weight
1. Likely warhead size as determined by radar cross-section	______
2. BM's range from the defended area	______
3. Number of interceptors that we can launch at a BM before it detonates	______
4. Predicted point of BM detonation	______
5. Number of batteries able to fire at the BM	______

B. List any criterion you feel should be considered that is not on A above and assign a weight as though the criterion was listed.

1. ______

2. ______

3. ______

C. Years service in Air Defense: ______

D. Comments:

APPENDIX B

GLOSSARY OF FORTRAN NAMES

The following is a list of Fortran variable names in order of their appearance in the example program.

V1	THE YIELD'S OF THE BALLISTIC MISSILE IN KILOTON E. G., 400. THE ARRAY IS SORTED BY "SHSORT"
V2	THE TIME BEFORE IMPACT IN SECONDS, E.G., 240. THE ARRAY OF NUMBERS TO BE SORTED BY "SHSORT".
V3	THE VALUE OF THE IMPACT POINT ON A SCALE OF 1, 2, OR 3, E. G., 1. THE ARRAY OF NUMBERS TO BE SORTED. THE ARRAY IS SORTED BY "SHSORT".
NBM	NUMBER OF ENEMY BALLISTIC MISSILES, E. G., 6.
BM	NUMBER OF VALUES IN ARRAYS "V1", "VA1", ETC.
VA1(1)	ARRAY, DIMENSIONED AT LEAST NBM IN CALLING PROGRAM, TO BE FILLED WITH INTEGERS FROM 1 TO BM. AFTER EXIT FROM SUBROUTINE "SHSORT", VA1(1) WILL CONTAIN THE ORIGINAL INDEX OF THE LARGEST ELEMENT OF "V1". VA (2) WILL CONTAIN THE ORIGINAL INDEX OF THE NEXT-TO-LARGEST ELEMENT OF "V1", etc. VA1(BM) WILL CONTAIN THE ORIGINAL INDEX OF THE SMALLEST ELEMENT OF "V1".
VA2(1)	SAME AS VA1
VS3(1)	SAME AS VA1
WEIG(1)	SAME AS VA1
WNN(1)	SAME AS VA1
XI(1)	CONTAINS THE ORIGINAL VALUE OF THE V1(1)TH VARIABLE AFTER IT HAS BEEN SORTED
X2(1)	SAME AS X1(1)
X3(1)	SAME AS X1(1)

SHSORT(V1,VA1,NBM) USED TO SORT, IN DECENDING ORDER, AN ARRAY OF SINGLE PRECISION REAL NUMBERS AND TO PRODUCE AN ARRAY OF INDEXES SO USER CAN RE-ORDER OTHER CORRESPONDING INFORMATION ACCORDING TO DECENDING VALUES OF "V1".

D1	THE ORIGINAL VALUE, X1, MINUS THE SMALLEST ELEMENT OF V1, ALL DIVIDED BY THE RANGE, D1.
D2, A2	SAME AS D1 and A1
D3, A3	SAME AS D1 and A1
WN(1)	EQUATION 3
W(1)	EQUATION 1
XW(1)	CONTAINS THE ORIGINAL VALUE OF W(1) BEFORE IT IS REORDERED BY "SHSORT"
XWN(1)	CONTAINS THE ORIGINAL VALUE OF WN(1) BEFORE IT IS REORDERED BY "SHSORT"

APPENDIX C

UNRANKED RAW CRITERIA DATA

	C1	C2	C3	C4	C5
AWC					
1	50	25	35	100	25
2	50	95	75	100	60
3	70	80	20	90	20
4	50	75	40	90	40
5	10	50	75	60	65
6	50	100	80	90	70
7	75	90	70	50	45
8	80	90	20	70	10
9	100	30	0	60	0
C & GS					
1	40	10	90	60	80
2	10	85	90	90	30
3	10	20	30	15	25
4	60	40	100	50	70
5	70	100	50	75	40
6	10	100	80	90	30
7	10	80	79	90	78
8	60	100	80	90	70
9	90	95	91	85	80
10	40	90	75	80	60
11	90	80	60	100	50
12	100	50	0	75	0
13	25	75	50	75	40
14	50	70	40	100	30
15	70	80	50	75	65

)RTRAN PROGRAM

```
PROGRAM SHOWING USE    D COMPARISON OF EQ. 1 AND EQ. ?
      DIMENSION V1(    ,V2(30),V3(30),VA1(30),VA2(3  ,VA3(30)
     *,W(30),WEIG(3    NEWEIG(30),WN(30),WNN(30),WEWN(30),
     *X1(30),X2(30),    30),XW(30),XWN(30)
      INTEGER VA1,V     A3,WEIG,WNN,BM
     YIELD OF WEAPON IN KILOTONS
        T  V1/400  ,  C.,275.,300.,200.,250./
V2:   TIME BEFORE IMPACT OF BM
      DATA V2/24  .,2 C.,240.,260.,350.,350./
V3:   VALUE OF IMPACT AREA
      DATA V3/1.,2.,  .,3.,3.,1./
      NBM=6
      BM=6
      DO 10 I=1,NBM
      VA1(I)=I
      VA2(I)=I
      VA3(I)=I
      WEIG(I)=I
   10 WNN(I)=I
      DO 11 I=1,NBM
      X1(I)=V1(I)
      X2(I)=V2(I)
   11 X3(I)=V3(I)
      CALL SHSORT(V1,VA1,NBM)
      CALL SHSORT(V2,VA2,NBM)
      CALL SHSORT(V3,VA3,NBM)
      DO 12 I=1,NBM
      D1=V1(1)-V1(BM)
      IF(D1) 20,20,21
   20 A1=0.0
      GO TO 22
   21 A1=(X1(I)-V1(BM))/D1
   22 D2=V2(1)-V2(BM)
      IF(D2) 24,24,25
   24 A2=0.0
      GO TO 26
   25 A2=(X2(I)-V2(BM))/D2
   26 D3=V3(1)-V3(BM)
      IF(D3) 28,28,29
   28 A3=0.0
      GO TO 30
   29 A3=(X3(I)-V3(BM))/D3
EQUATION 3
   30 WN(I)=.25*A1 -.37*A2 +.38*A3
EQUATION 1
   12 W(I)=.25*X1(I)-.37*X2(I)+.33*X3(I)
      DO 50 I=1,NBM
      XW(I)=W(I)
   50 XWN(I)=WN(I)
      CALL SHSORT(W,WEIG,NBM)
      CALL SHSORT(WN,WNN,NBM)
      WRITE  ,40)
   40 FORMAT(//////,10X,'VALUE EQ. 1',6X,'VALUE EQ. 3',7X,
     *'V1',6X,'V2',6X,'V3',//)
      DO 41 I=1,NBM
      WRITE(6,42) XW(I),XWN(I),X1(I),X2(I),X3(I)
   42 FORMAT(9X,F9.2,8X,F9.2, 9X,F4.0,4X,F4.0,4X,F4.0)
   41 CONTINUE
      WRITE(6,43)
   43 FORMAT(///,20X,'BM',3X,'EQ. 1 ORDERED',7X,
     *'EQ. 3 ORDERED',///)
      DO 44 I=1,NBM
      WRITE(6,45) I,W(I),WN(I)
   45 FORMAT(20X,I2,2X,F10.2,9X,F10.2)
   44 CONTINUE
      STOP
      END
```

```
      SUBROUTINE SHSORT(A,KEY,/N/)
      DIMENSION A(N),KEY(N)
      M1=1
    6 M1=M1*2
      IF(M1-N) 6,6,8
    8 M1=M1/2-1
      MM=MAX0(M1/2,1)
      GO TO 21
   20 MM=MM/2
      IF(MM)100,100,21
   21 K=N-MM
   22 DO 1 J=1,K
      II=J
   11 IM=II+MM
      IF(A(IM)-A(II)) 1,1,30
   30 TEMP=A(II)
      IT=KEY(II)
      A(II)=A(IM)
      KEY(II)=KEY(IM)
      A(IM)=TEMP
      KEY(IM)=IT
      II=II-MM
      IF(II) 1,1,11
    1 CONTINUE
      GO TO 20
  100 RETURN
      END
```

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13. ABSTRACT

This paper is concerned with those criteria by which enemy ballistic missiles are threat ordered. A questionnaire was sent to Army Air Defense Officers at two senior service schools to elicit their opinions. The questionnaires were analyzed using Kendall's coefficient of concordance. Relative weights were determined for each criteria.

A linear model was developed using selected criterion and their relative weights. The number of interceptors was fixed. A sample problem shows how one might use the linear model to determine which ballistic missile to engage first. The number of interceptors to be fired at a threat is discussed and a simple non-linear program is formulated where the number of interceptors can vary.

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